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ON THE ITO FORMULA IN A BANACH SPACE

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Abstract. If $(W_t)_{t\in[0,1]}$ is a Wiener process in an arbitrary separable Banach space X, $\psi : [0,1]\times X \to Y$ is a continuous function with values in another separable Banach space, and ψ has continuous Frechet derivatives ψ'_t, ψ'_x and ψ''_{xx}, then the Ito formula is obtained for $\psi(t, W_t)$.

The method is based on the concept of covariance operator and a special construction of the Ito stochastic integral.

Let X be a real separable Banach space, X^* its dual, $(\Omega, \mathcal{B}, P)$ a probability space, $\mathcal{B}(X)$ the Borel σ-algebra of X. A random element (measurable map) $\xi : \Omega \to X$ is called Gaussian random element, if $\langle \xi, x^*\rangle$ is a Gaussian random variable for all $x^* \in X^*$. $\mu_\xi(B) \equiv P\{\xi \in B\}$, $B \in \mathcal{B}(X)$ is a distribution of ξ. μ_ξ is uniqually determined by the mean $E\xi = \int\limits_\Omega \xi(\omega)dP$ and the covariance operator $R : X^* \to X$, $\langle Rx^*, y^*\rangle = E(\langle \xi - E\xi, x^*\rangle\langle \xi - E\xi, y^*\rangle$. R is a simmetric positive linear operator. Covariance operators of Gaussian measures are called Gaussian covariances.

A family of random elements $(W_t)_{t\in[0,1]}, W_t : \Omega \to X$ is called a (homogeneous) Wiener process (with values in X), if 1) $W_0 = 0$ almost surely (a.s); 2) $W_{t_{i+1}} - W_{t_i}$ $(i = 0, 1, \cdots, n-1)$ are independent random elemensts for every $0 \le t_0 < t_1 < \cdots < t_n \le 1$; 3) for every $t \in [0,1]$, W_t is a Gaussian random element with covariance operator tR, where $R : X^* \to X$ is a fixed Gaussian covariance; 4) $(W_t)_{t\in[0,1]}$ has a.s. continuous sample paths.

If X is a finite-dimensional Hilbert space and R is the identity operator, then our definition of a Wiener process coincides with the usual definition of a finite-dimensional Wiener process. When X is an infinite-dimensional Hilbert space, then no Gaussian measure with the identity covariance operator exists. Therefore, no Wiener process exists for which R is the identity operator. Our definition is a direct extension of the definition of a Wiener process for the Hilbert space case ([1], p. 113). L. Gross [2] gave a definition of a Wiener process in a Banach spase by introducing a special notion of a measurable norm.

Let $(\mathcal{F}_t)_{t\in[0,1]}$, $\mathcal{F}_t \subset \mathcal{B}$ be a family of increasing σ-algebras (if $0 \le t_1 < t_2 \le 1$ then $\mathcal{F}_{t_1} \subset \mathcal{F}_{t_2}$), which is adapted to the Wiener process $(W_t)_{t\in[0,1]}$, i.e.: a)

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W_t is $\mathcal{F}_t$-measurable for all $t \in [0,1]$; b) $W_s - W_t$ is independent of $\mathcal{F}_t$ for $0 \le t < s \le 1$. For many purposes we need $\mathcal{F}_0$ to contain all P-null sets in $\mathcal{B}$.

Definition 1. A function $\varphi : [0,1] \times \Omega \to X^*$ is called nonanticipating with respect to $(\mathcal{F}_t)_{t\in[0,1]}$ if the function $(t,\omega) \to \langle x, \varphi(t,\omega)\rangle$ from $([0,1] \times \Omega, \mathcal{B}[0,1] \times \mathcal{B})$ into $(R^1, \mathcal{B}(R^1))$ is measurable for all $x \in X$, and the function $\omega \to \langle x, \varphi(t,\omega)\rangle$ from $(\Omega, \mathcal{B})$ into $(R^1, \mathcal{B}(R^1))$ is $\mathcal{F}_t$-measurable for all $t \in [0,1]$.

By $G_R(X^*)$ we define the class of nonanticipating function φ, for which $P_R(\varphi) = (\int_0^1 \int_\Omega \langle R\varphi(t,\omega), \varphi(t,\omega)\rangle \, dt\, dp)^{1/2} < \infty$. $G_R(X^*)$ is a linear space and P_R is a pseudonorm in it.

Proposition 1. *If $\varphi : [0,1] \times \Omega \to X^*$ is nonanticipating and for all $x \in X$ $\int_0^1 \int_\Omega \langle x, \varphi(t,\omega)\rangle^2 \, dt\, dp < \infty$, then $\varphi \in G_R(X^*)$.*

Proof. Consider the linear operator $T : X \to L_2([0,1]\times\Omega, \mathcal{B}[0,1]\times\mathcal{B}, \lambda\times P)$ (by λ we denote the Lebesgue measure on $[0,1]$), $Tx = \langle x, \varphi(t,\omega)\rangle$; by the closed graph theorem it is easy to see that ([4], p. 95) T is a bounded operator, i.e., $\sup_{\|x\|\le 1} \int_0^1 \int_\Omega \langle x,\, \varphi(t,\omega)\rangle^2 \, dt\, dp < \infty$. We will repeatedly use the following

Theorem (Kwapień S., Szymanski B. [3], [4] p. 262). *Let $R : X^* \to X$ be a Gaussian covariance, then there exists $(x_n^*)_{n\in N} \subset X^*$ and $(x_n)_{n\in N} \subset X$ such that $\langle x_n^*, x_k\rangle = \delta_{n_k}$, $Rx^* = \sum_{k=1}^{\infty} \langle x_k, x^*\rangle x_k$, $x^* \in X^*$ and $\sum_{k=1}^{\infty} \|x_k\|^2 < \infty$.*

By Kwapień–Szymanski's theorem we have

$$\int_0^1 \int_\Omega \langle R\varphi(t,\omega),\, \varphi(t,\omega)\rangle \, dt\, dp = \int_0^1 \int_\Omega \sum_{k=1}^{\infty} \langle x_k, \varphi(t,\omega)\rangle^2 \, dt\, dp$$

$$= \sum_{k=1}^{\infty} \|x_k\|^2 \int_0^1 \int_\Omega \Big\langle \frac{x_k}{\|x_k\|}, \varphi(t,\omega)\Big\rangle^2 \, dt\, dp$$

$$\le \sum_{k=1}^{\infty} \|x_k\|^2 \sup_{\|x\|\le 1} \int_0^1 \int_\Omega \langle x, \varphi(t,\omega)\rangle^2 \, dt\, dp < \infty. \qquad \square$$

If $\varphi \in G_R(X^*)$ is a step-function $\varphi(t,\omega) = \sum_{i=0}^{n-1} \varphi_{t_i}(\omega)\chi_{(t_i,t_{i+1}]}$, $0 = t_0 < \cdots < t_n = 1$, $\varphi_{t_i} : \Omega \to X^*$, $i = 0, \dots, n-1$, then the stochastic integral of φ with respect to $(W_t)_{t\in[0,1]}$ is naturally defined by the equality

$$\int_0^1 \varphi(t,\omega)\, dW_t = \sum_{i=0}^{n-1} \Big\langle \varphi_{t_i}(\omega), W_{t_{i+1}} - W_{t_i} \Big\rangle.$$

The following lemma is true.

Lemma 1 ([5]). *For an arbitrary $\varphi \in G_R(X^*)$ there exists a sequence of step-functions $(\varphi_n)_{n\in N} \subset G_R(X^*)$ such that $\varphi_n \xrightarrow{P_R} \varphi$ and $\int_0^1 \varphi_n \, dW_t$ converges in $L_2(\Omega, \mathcal{B}, P)$ as $n \to \infty$.*

Definition 2 ([5]). Let $\varphi \in G_R(X^*)$ and $(\varphi_n)_{n\in N} \subset G_R(X^*)$ be step-functions such that $\varphi_n \xrightarrow{P_R} \varphi$ and $\int_0^1 \varphi_n \, dW_t$ converges in $L_2(\Omega, \mathcal{B}, P)$. The limit of the sequence $\int_0^1 \varphi_n \, dW_t$ is called the stochastic integral of a random function $\varphi \in G_R(X^*)$ with respect to the Wiener process $(W_t)_{t\in[0,1]}$ and is denoted by $\int_0^1 \varphi \, dW_t$.

It follows from Lemma 1 that for an arbitrary $\varphi \in G_R(X^*)$ there exists the stochastic integral $\int_0^1 \varphi \, dW_t$. If we denote by $G(X^*)$ the class of nonanticipating functions φ such that $\int_0^1 \int_\Omega \langle x, \varphi(t,\omega)\rangle^2 \; dt \, dp < \infty$ for all $x \in X$, then by Proposition 1 $G(X^*) \subset G_R(X^*)$ for all Gaussian covariances R.

As in the one-dimensional case, we can define the stochastic integral of nonanticipating functions $\varphi : [0,1] \times \Omega \to X^*$ such that

$$P\Big\{\omega : \quad \int_0^1 \langle R\varphi(t,\omega), \varphi(t,\omega)\rangle \, dt < \infty\Big\} = 1.$$

The class of such functions is denoted by $\mathcal{L}_R(X^*)$.

Lemma 2. *Let $\varphi \in \mathcal{L}_R(X^*)$. Then there exists a sequence $(\varphi_n)_{n\in N} \subset G_R(X^*)$ such that*

$$P\Big\{\omega : \quad \int_0^1 \Big\langle R\varphi(t,\omega) - \varphi_n(t,\omega), \varphi(t,\omega) - \varphi_n(t,\omega)\Big\rangle \, dt > \varepsilon\Big\} \to 0$$

for all $\varepsilon > 0$, as $n \to \infty$.

Proof. Denote by

$$\tau_n(\omega) = \begin{cases} \inf\Big\{t \le 1, \ \int_0^t \langle R\varphi(s,\omega), \varphi(s,\omega)\rangle \, ds \ge N\Big\} \\ 1, \quad \text{if} \quad \int_0^1 \langle R\varphi(s,\omega), \varphi(s,\omega)\rangle \, ds < N \end{cases},$$

$\varphi_N(s,\omega) \equiv \varphi(s,\omega)\cdot\chi_{[s\le\tau_n(\omega)]}$. τ_n is a Markov moment with respect to the family $(\mathcal{F}_t)_{t\le 1}$. Therefore $\varphi_N(s,\omega)$, $N=1,2,\dots$, are nonanticipating functions, and

$$\int_0^1\int_\Omega \langle R\varphi_N(s,\omega),\varphi_N(s,\omega)\rangle\,dt\,dp<\infty.$$

Now

$$P\Big\{\omega:\ \int_0^1\Big\langle R\varphi(t,\omega)-\varphi_N(t,\omega),\varphi(t,\omega)-\varphi_N(t,\omega)\Big\rangle\,dt>\varepsilon\Big\}$$

$$=P\Big\{\omega:\ \int_0^1\langle R\varphi(t,\omega),\varphi(t,\omega)\rangle\,dt>N\Big\}\to 0$$

as $N\to\infty$ for all $\varepsilon>0$. $\square$

For each t, $0\le t\le 1$ and $\varphi\in G_R(X^*)$ we define

$$\int_0^t\varphi(s,\omega)\,dw_s\equiv\int_0^1\varphi(s,\omega)I_{[0,t]}^{(s)}\,dw_s.$$

As in the one-dimensional case, we can prove that $\int_0^t\varphi(s,\omega)dw_s$, $0\le t\le 1$, has a modification with a.s. continuous sample paths.

The following lemma is true.

Lemma 3. *Let $\varphi\in G_R(X^*)$. Then for all $N>0$ and $c>0$,*

$$P\Big\{\sup_{0\le t\le 1}\Big|\int_0^t\varphi(s,\omega)\,dw_s\Big|>c\Big\}\le\frac{N}{c^2}+P\Big\{\int_0^1\langle R\varphi(s,\omega),\varphi(s,\omega)\rangle\,ds>N\Big\}.$$

The proof is analogous to that of the lemma in the one-dimensional case.

Now we can define the stochastic integral of $\varphi\in\mathcal{L}_R(X^*)$. Let $(\varphi_n)_{n\in N}\subset G_R(X^*)$ be a sequence, such that for all $\varepsilon>0$ $P\{\int_0^1\langle R\varphi_n-\varphi,\varphi_n-\varphi\rangle\,dt>\varepsilon\}\to 0$ as $n\to\infty$. Then as in the one-dimensional case we can prove, that the sequence $(\int_0^1\varphi_n(t,\omega)\,dw_t)_{n\in N}$ converges in probability. We define

$$\int_0^1\varphi(t,\omega)\,dw_t=\lim_{n\to\infty}\int_0^1\varphi_n(t,\omega)\,dw_t$$

and

$$\int_0^t\varphi(s,\omega)\,dw_s=\int_0^1\varphi(s,\omega)I_{[0,t]}^{(s)}\,dw_s.$$

Let Y be another separable Banach space, Y^* its dual, $L(X,Y)(L(Y^*,X^*))$ the Banach space of linear bounded operators from X to Y (from Y^* to X^*).

Definition 3. A function $\varphi : [0,1] \times \Omega \to L(X,Y)$ is called nonanticipating with respect to $(\mathcal{F}_t)_{t\in[0,1]}$, if for all $x \in X$ and $y^* \in Y^*$ the real-valued function $(t,\omega) \to \langle \varphi(t,\omega)x, y^*\rangle$ is measurable and for all $t \in [0,1]$, $\omega \to \langle \varphi(t,\omega)x, y^*\rangle$ is a $\mathcal{F}_t$-measurable random variable. By $G_R(L(X,Y))$ we denote the class of all nonanticipating functions $\varphi : [0,1] \times \Omega \to L(X,Y)$, such that for all $y^* \in Y^*$ $\sigma_R^2(y^*) = \int_0^1 \int_\Omega \langle \varphi(t,\omega) R\varphi^*(t,\omega)y^*, y^*\rangle \, dt\, dp < \infty$, where $\varphi^*(t,\omega)$ is the dual operator of $\varphi(t,\omega)$ and $R : X^* \to X$ is a Gaussian covariance. $G(L(X,Y))$ stends for the class of nonanticipating functions $\varphi : [0,1] \times \Omega \to L(X,Y)$ such that $\int_\Omega \int_0^1 \langle \varphi(t,\omega)x, y^*\rangle^2 \, dt\, dp < \infty$ for all $x \in X$ and $y^* \in Y^*$.

Proposition 2. *If $R : X^* \to X$ is a Gaussian covariance then $G(L(X,Y)) \subset G_R(L(X,Y))$.*

The proof is analogous to the that of Proposition 1.

Let $\varphi \in G_R(L(X,Y))$ and $y^* \in Y^*$. $\varphi^* y^*$ maps $[0,1] \times \Omega$ into X^* and $\varphi^* y^* \in G_R(X^*)$. Therefore we can define the stochastic integral $\int_0^1 \varphi^* y^* \, dW_t$. Consider the map $T_\varphi : Y^* \to L_2(\Omega, \mathcal{B}, P)$ $T_\varphi y^* = \int_0^1 \varphi^* y^* \, dW_t$. T_φ is a linear continuous map, i.e., it is a random linear function (RLF).

Definition 4 ([5]). Let $\varphi \in G_R(L(X,Y))$. The linear continuous map (RLF) $T_\varphi : Y^* \to L_2(\Omega, \mathcal{B}, P)$, defined by $T_\varphi y^* = \int_0^1 \varphi^* y^* \, dW_t$, $y^* \in Y^*$, is called the generalized stochastic integral of operator-valued random function φ with respect to $(W_t)_{t\in[0,1]}$.

Definition 5 ([5]). Let $\varphi \in G_R(L(X,Y))$. We say that a random element $\xi : \Omega \to Y$ (if such an element exsists) is the stochastic integral of φ with respect to a Wiener process $(W_t)_{t\in[0,1]}$ if $\langle \xi, y^*\rangle = T_\varphi y^*$ a.e. for all $y^* \in Y^*$, and write $\xi = \int_0^1 \varphi \, dW_t$.

We can define the generalized stochastic integral of a nonanticipating function $\varphi : [0,1] \times \Omega \to L(X,Y)$, such that

$$P\Big\{ \int_0^1 \Big\langle R\varphi^*(t,\omega)y^*, \varphi^*(t,\omega)y^* \Big\rangle \, dt < \infty \Big\} = 1$$

for all $y^* \in Y^*$. The class of such functions is denoted by $\mathcal{L}_R(L(X,Y))$. Define the linear operator $T_\varphi : Y^* \to L_0(\Omega, \mathcal{B}, P)$, $T_\varphi y^* = \int_0^1 \varphi^*(t,\omega) y^* \, dW_t$, called the generalized stochastic integral of φ. A random element $\xi : \Omega \to Y$ (if it exists) such that $T_\varphi y^* = \langle \xi, y^*\rangle$ a.e. for all $y^* \in Y^*$ is said to be the stochastic integral of φ and we write $\xi = \int_0^1 \varphi \, dW_t$. For each t, $0 \le t \le 1$ and $\varphi \in \mathcal{L}_R(L(X,Y))$, we denote the generalized stochastic integral $T^t_\varphi y^* = \int\limits_0^1 \varphi^*(s,\omega) y^* I_{[0,t]}(s)\, dW_s$ and the stochastic integral $\xi_t \equiv \int\limits_0^t \varphi(s,\omega)\, dW_s$, $\langle \xi_t, y^*\rangle = T^t_\varphi y^*$, $y^* \in Y^*$.

Thus the generalized stochastic integral exists for all $\varphi \in \mathcal{L}_R((L(X,Y))$, but the stochastic integral $\int\limits_0^1 \varphi(t,\omega)\, dW_t$ does not always exist. The generalized stochastic integral, as an RLF, induces a cylindrical measure on Y which cannot always be extended to a countably additive measure on the Borel σ-algebra $\mathcal{B}(Y)$. Thus, the question of the existence of a stochastic integral is reduced to the problem on extending of a cylindrical measure to a countably additive measure or, equivalently, to the problem of decomposability of an RLF.

Recall that in the finite-dimensional case, the Ito stochastic integral of a step function is defined naturally while the stochastic integral of nonanticipating function φ is defined as a limit of stochastic integrals of step functions $(\varphi_n)_{n\in N}$ converging to φ. This method makes it possible to define the stochastic integral in a Hilbert space (see [7], [8]) and in a Banach space, where the square of the norm has a derivative satisfyng the Lipschitz condition (see [9]).

The Ito formula. Let $(W_t)_{t\in[0,1]}$ be a Wiener process in X, $R : X^* \to X$ the covariance operator of W_1, $Rx^* = \sum\limits_{k=1}^{\infty} \langle a_k, x^*\rangle a_k$ its any representation by the series, $(a_k)_{k\in N} \subset X$. The following theorem is true.

Theorem 1 (The Ito formula). *Let $\psi : [0,1]\times X \to Y$ be a continuous function with continuous Frechet derivatives $\psi'_t : [0,1]\times X \to Y$, $\psi'_x : [0,1]\times X \to L(X,Y)$ and $\psi''_{xx} : [0,1]\times X \to L(X, L(X,Y))$. Then*

$$\psi(t,W_t) = \psi(0,0) + \int\limits_0^t \psi'_t(s,W_s)\, ds + \int\limits_0^t \psi'_x(s,W_s)\, dW_s + \frac{1}{2}\int\limits_0^t \Big(\sum\limits_{i=1}^{\infty} \psi''_{xx}(s,W_s) a_i a_i\Big)\, ds.$$

Proof. As in the one-dimensional case, for all fixed t, denote $l \equiv [2^n\cdot t]$, $\triangle W \equiv W_{\frac{k}{2^n}} - W_{\frac{k-1}{2^n}}$, $n = 1,2,\dots$ and use the Taylor formula:

$$\begin{aligned}
&\psi(t,W_t) - \psi(0,0)\\
&= \sum_{k\le l}\Big[\psi\Big(\frac{k}{2^n}, W_{\frac{k}{2^n}}\Big) - \psi\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\Big] + \psi(t,W_t) - \psi\Big(\frac{l}{2^n}, W_{\frac{l}{2^n}}\Big)\\
&= \sum_{k\le l}\int\limits_0^1 \psi'_t\Big(\frac{k-1}{2^n} + \frac{s}{2^n}, W_{\frac{k-1}{2^n}}\Big)\cdot\frac{1}{2^n}\, ds + \sum_{k\le l}\psi'_x\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\Delta W\\
&\quad + \sum_{k\le l}\int\limits_0^1 (1-s)\psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}} + s\Delta W\Big)\Delta W\Delta W\, ds
\end{aligned}$$

$$+\psi(t,W_t)-\psi\Big(\frac{l}{2^n},W_{\frac{l}{2^n}}\Big)=\sum_{k\le l}\psi_t'\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\cdot\frac{1}{2^n}$$

$$+\sum_{k\le l}\int_0^1\Big[\psi_t'\Big(\frac{k-1}{2^n}+\frac{s}{2^n},W_{\frac{k-1}{2^n}}\Big)-\psi_t'\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Big]\frac{1}{2^n}\,ds$$

$$+\sum_{k\le l}\psi_x'\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Delta W_k+\frac{1}{2}\sum_{k\le l}\Big(\sum_i\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_i\Big)\frac{1}{2^n}$$

$$+\sum_{k\le l}\Big[\int_0^1(1-s)\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}+s\Delta W\Big)\Delta W\Delta W\,ds$$

$$-\frac{1}{2}\,\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Delta W\Delta W\Big]+\frac{1}{2}\sum_{k\le l}\Big[\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Delta W\Delta W$$

$$-\frac{1}{2^n}\sum_i\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_i\Big]+\psi(t,W_t)-\psi\Big(\frac{l}{2^n},W_{\frac{l}{2^n}}\Big)$$

$$=\sum_{k\le l}\psi_t'\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\cdot\frac{1}{2^n}+\sum_{k\le l}\psi_x'\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Delta W$$

$$+\frac{1}{2}\sum_{k\le l}\Big(\sum_i\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_i\Big)+A_n+B_n+C_n+\delta_n, \tag{1}$$

where

$$A_n\equiv\sum_{k\le l}\int_0^1\Big[\psi_t'\Big(\frac{k-1}{2^n}+\frac{s}{2^n},W_{\frac{k-1}{2^n}}\Big)-\psi_t'\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Big]\frac{1}{2^n}\,ds,$$

$$B_n\equiv\sum_{k\le l}\Big[\int_0^1(1-s)\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}+s\Delta W\Big)\Delta W\Delta W\,ds$$

$$-\frac{1}{2}\,\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Delta W\Delta W\Big],$$

$$C_n\equiv\frac{1}{2}\sum_{k\le l}\Big[\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Delta W\Delta W-\frac{1}{2^n}\sum_i\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_i\Big],$$

$$\delta_n\equiv\psi(t,W_t)-\psi\Big(\frac{l}{2^n},W_{\frac{l}{2^n}}\Big)\to 0\quad\text{a.s. as}\quad n\to\infty.$$

Denote by

$$\alpha_n(\omega)\equiv\sup_{k\le l}\Big\|\psi_t'\Big(\frac{k-1}{2^n}+\frac{s}{2^n},W_{\frac{k-1}{2^n}}\Big)-\psi_t'\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Big\|$$

and

$$\beta_n(\omega)\equiv\sup_{k\le l}\Big\|\int_0^1(1-s)\Big[\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}+s\Delta W\Big)-\psi_{xx}''\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)\Big]\,ds\Big\|.$$

By the continuity of sample paths of the Wiener process and that of the functions ψ'_t and ψ''_{xx} we conclude, that $\alpha_n \to 0$ a.s. and

$$\beta_n \leq \frac{1}{2} \sup_{\substack{k\leq l\\ 0\leq s\leq 1}} \Big\|\psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}} + s\Delta W\Big) - \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\Big\| \to 0$$
$$\text{a.s. as} \quad n \to \infty.$$

Further we have $\|A_n(\omega)\| \leq \alpha_n(\omega) \cdot t \to 0$ a.s. and

$$\|B_n\| \leq \sum_{k\leq l} \Big\| \int_0^1 (1-s)\Big[\psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}} + s\Delta W\Big)$$
$$- \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\Big]\, ds\Big\| \, \|\Delta W\|^2 \leq \beta_n \sum_{k\leq l} \|\Delta W\|^2.$$

Now we can show that $\sum\limits_k \|W_{\frac{k}{2^n}} - W_{\frac{k-1}{2^n}}\|^2 \xrightarrow{P} tE\|W_t\|^2$ as $n \to \infty$. Indeed,

$$E\Big[\sum_{k\leq l}\Big(\|W_{\frac{k}{2^n}} - W_{\frac{k-1}{2^n}}\|^2 - \frac{1}{2^n}\, E\|W_1\|^2\Big)\Big]^2$$
$$= \sum_{k\leq l} E\Big(\|W_{\frac{k}{2^n}} - W_{\frac{k-1}{2^n}}\|^2 - \frac{1}{2^n}\, E\|W_1\|^2\Big)^2$$
$$= \sum_{k\leq l}\Big(E\Big(\|W_{\frac{k}{2^n}} - W_{\frac{k-1}{2^n}}\|\Big)^4 - \frac{1}{2^{2n}}\,(E\|W_1\|^2)^2\Big)$$
$$\leq 2^n\,\frac{1}{2^{2n}}\Big(E\|W_1\|^4 - (E\|W_1\|^2)^2\Big) = \frac{1}{2^n}\Big[E\|W_1\|^4 - (E\|W_1\|^2)^2\Big] \to 0$$

as $n \to \infty$, i.e., $\sum\limits_{k\leq l} \|W_{\frac{k}{2^n}} - W_{\frac{k-1}{2^n}}\|^2 \xrightarrow{P} tE\|W_1\|^2$, therefore, $\|B_n\| \xrightarrow{P} 0$ as $n \to \infty$.

Now we show, that $\langle C_n, y^*\rangle \xrightarrow{P} 0$ for all $y^* \in Y^*$ as $n \to \infty$. Denote by $\chi^N_k \equiv \chi_{\{\max\limits_{i\leq k}\|W_{i/2n}\|\leq N\}}$. Then χ^N_k is $\mathcal{F}_{\frac{k}{2^n}}$-measurable.

$$\langle C_n, y^*\rangle = \frac{1}{2}\sum_{k\leq l}\chi^N_{k-1}\Big(\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\Delta W\Delta W, y^*\Big\rangle$$
$$-\frac{1}{2^n}\sum_i\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_i, y^*\Big\rangle\Big)$$
$$+\frac{1}{2}\sum_{k\leq l}(1-\chi^N_{k-1})\Big(\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\Delta W\Delta W, y^*\Big\rangle$$
$$-\frac{1}{2^n}\sum_i\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_i, y^*\Big\rangle\Big),$$

$$P\Big\{\sum_{k\leq l}(1-\chi^N_{k-1})\Big(\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\Delta W\Delta W, y^*\Big\rangle$$

$$-\frac{1}{2^n}\sum_i \Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_i, y^*\Big\rangle\Big) \neq 0\Big\}$$

$$\leq P\Big\{\sum_{k\leq l}(1-\chi^N_{k-1}) \neq 0\Big\} \leq P\Big\{\sup_{0\leq t\leq 1}\|W_t\| > N\Big\} \to 0 \quad \text{as} \quad N\to\infty.$$

Thus it is enough to show, that

$$\langle \widetilde{C}_n, y^*\rangle \equiv \frac{1}{2}\sum_{k\leq l}\chi^N_{k-1}\Big(\Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\Delta W\Delta W, y^*\Big\rangle$$
$$-\frac{1}{2^n}\sum_i \Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_i, y^*\Big\rangle\Big)$$

converges in probability to 0, as $n\to\infty$.

Let now $(a_k)_{k\in N}\subset X$ be such that there exists $(a^*_k)_{k\in N}$, $\langle a_k, a^*_j\rangle = \delta_{kj}$. Then in the representation of the Wiener process $W_t = \sum_{k=1}^{\infty} a_k\zeta^k_t$ (see [6]), $(\zeta^k_t)_{t\in[0,1]}$, $k=1,2,\ldots$, are independent real valued Wiener processes, $\langle W_t, a^*_k\rangle = \zeta^k_t$. Hence, $(\mathcal{F}_t)_{t\in[0,1]}$, is adapted to the Wiener processes $(\zeta^k_t)_{t\in[0,1]}$, $k=1,2,\ldots$. Let also $\sum_{k=1}^{\infty}\|a_k\|^2 < \infty$ (this is possible by Kwapień–Szimansky's theorem). Denote $\Delta\zeta^i_k \equiv \zeta^i_{\frac{k}{2^n}} - \zeta^i_{\frac{k-1}{2^n}}$. Now

$$E\langle \widetilde{C}_n, y^*\rangle^2 = \frac{1}{4}E\bigg(\sum_{k\leq l}\chi^N_{k-1}\Big(\Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\Delta W\Delta W, y^*\Big\rangle$$
$$-\frac{1}{2^n}\sum_i \Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_i, y^*\Big\rangle\Big)\bigg)^2$$
$$= \frac{1}{4}E\bigg(\sum_{k\leq l}\chi^N_{k-1}\Big(\sum_{i,j}\Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_j, y^*\Big\rangle\Delta\zeta^i_k\Delta\zeta^J_k$$
$$-\frac{1}{2^n}\sum_i \Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_i, y^*\Big\rangle\Big)\bigg)^2$$
$$= \frac{1}{4}E\bigg(\sum_{k\leq l}\chi^N_{k-1}\Big(\sum_i \Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_i, y^*\Big\rangle\Big(\Delta\zeta^{i^2}_k - \frac{1}{2^n}\Big)$$
$$+\sum_{i\neq j}\Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_j, y^*\Big\rangle\Delta\zeta^i_k\Delta\zeta^j_k\Big)\bigg)^2$$
$$\leq \frac{1}{2}E\Big[\sum_{k\leq l}\chi^N_{k-1}\Big(\sum_i \Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_i, y^*\Big\rangle\Big(\Delta\zeta^{i^2}_k - \frac{1}{2^n}\Big)\Big)\Big]^2$$
$$+\frac{1}{2}E\Big[\sum_{k\leq l}\chi^N_{k-1}\sum_{i\neq j}\Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_i a_j, y^*\Big\rangle\Delta\zeta^i_k\Delta\zeta^J_k\Big]^2.$$

Consider the first term of the sum:

$$E\Big[\sum_{k\le l}\chi^N_{k-1}\Big(\sum_i\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_i,y^*\Big\rangle\Big(\Delta\zeta_k^{i^2}-\frac{1}{2^n}\Big)\Big)\Big]^2$$

$$=\sum_{k\le l}\sum_{k'\le l}\sum_i\sum_{i'}E\Big[E\Big(\chi^N_{k-1}\chi^N_{k'-1}\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_i,y^*\Big\rangle$$

$$\times\Big\langle\psi''_{xx}\Big(\frac{k'-1}{2^n},W_{\frac{k'-1}{2^n}}\Big)a_{i'}a_{i'},y^*\Big\rangle$$

$$\times\Big(\Delta\zeta_k^{i^2}-\frac{1}{2^n}\Big)\Big(\Delta\zeta_{k'}^{i'^2}-\frac{1}{2^n}\Big)\Big/\mathcal{F}_{\max(\frac{k-1}{2^n},\frac{k'-1}{2^n})}\Big)\Big]$$

$$=\sum_{k\le l}E\chi^N_{k-1}\sum_i\sum_{i'}E\Big(\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_i,y^*\Big\rangle$$

$$\times\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a'_ia'_i,y^*\Big\rangle\Big(\Delta\zeta_k^{i^2}-\frac{1}{2^n}\Big)\Big(\Delta\zeta_k^{i'^2}-\frac{1}{2^n}\Big)\Big/\mathcal{F}_{\frac{k-1}{2^n}}\Big)$$

$$\le\sum_{k\le l}\sum_i\sup_{\substack{\|x\|\le N\\0\le t\le 1}}\|\psi''_{xx}(t,x)\|^2\|a_i\|^4\cdot\|y^*\|^2E\Big(\Delta\zeta_k^{i^2}-\frac{1}{2^n}\Big)^2$$

$$\le\sup_{\substack{\|x\|\le N\\0\le t\le 1}}\|\psi''_{xx}(t,x)\|^2\cdot\|y^*\|^2\cdot 2^n\cdot\frac{1}{2^{2n}}\sum_{i=1}^\infty\|a_i\|^4\cdot E(\gamma^2-1)^2=c\cdot\frac{1}{2^n}\to 0,$$

as $n\to\infty$, where γ is a standard Gaussian random variable.

Consider the second term of the sum:

$$E\Big(\sum_{k\le l}\chi^N_{k-1}\sum_{i\ne j}\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_j,y^*\Big\rangle\Delta\zeta_k^i\Delta\zeta_k^j\Big)^2$$

$$=\sum_{k\le l}\sum_{k'\le l}\sum_{i\ne j}\sum_{i'\ne j'}E\Big(E\Big(\chi^N_{k-1}\chi^N_{k'-1}\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_j,y^*\Big\rangle$$

$$\times\Big\langle\psi''_{xx}\Big(\frac{k'-1}{2^n},W_{\frac{k'-1}{2^n}}\Big)a'_ia'_j,y^*\Big\rangle\Delta\zeta_k^i\Delta\zeta_k^j\Delta\zeta_{k'}^{i'}\Delta\zeta_{k'}^{j'}\Big/\mathcal{F}_{\max(\frac{k-1}{2^n},\frac{k'-1}{2^n})}\Big)\Big)$$

$$=\sum_{k\le l}\sum_{i\ne j}\sum_{i'\ne j'}E\Big(E\Big(\chi^N_{k-1}\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_j,y^*\Big\rangle$$

$$\times\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a'_ia'_j,y^*\Big\rangle\Delta\zeta_k^i\Delta\zeta_k^j\Delta\zeta_k^{i'}\Delta\zeta_k^{j'}\Big/\mathcal{F}_{\frac{k-1}{2^n}}\Big)\Big)$$

$$=\sum_{k\le l}\sum_{i\ne j}E\Big(\chi^N_{k-1}\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_j,y^*\Big\rangle^2\Delta\zeta_k^{i^2}\Delta\zeta_k^{j^2}\Big)$$

$$+\sum_{k\le l}\sum_{i\ne j}E\Big(\chi^N_{k-1}\Big\langle\psi''_{xx}\Big(\frac{k-1}{2^n},W_{\frac{k-1}{2^n}}\Big)a_ia_j,y^*\rangle$$

$$\times\Big\langle \psi''_{xx}\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)a_j a_i, y^*\Big\rangle \Delta\zeta_k^{i^2}\Delta\zeta_k^{j^2}\Big)$$

$$\leq \sup_{\substack{\|x\|\leq N\\ 0\leq t\leq 1}} \|\psi''_{xx}(t,x)\|^2 \sum_i \|a_i\|^2 \sum_J \|a_j\|^2 \cdot 2^n \cdot \frac{1}{2^n}\,\frac{1}{2^n}\cdot \|y^*\|^2$$

$$+\sup_{\substack{\|x\|\leq N\\ 0\leq t\leq 1}} \|\psi''_{xx}(t,x)\|^2 \sum_i \|a_i\|^2 \sum_J \|a_j\|^2 \cdot \|y^*\|^2 \cdot \frac{1}{2^n}\,\frac{1}{2^n}\cdot 2^n \to 0 \ \text{ as } \ n\to\infty.$$

Therefore, for all $y^*\in Y^*$ we have $\langle \widetilde{C}_n, y^*\rangle \to 0$ in $L_2(\Omega,\mathcal{B},P)$, as $n\to\infty$.

The continuity of sample paths of the Wiener process $(W_t)_{t\in[0,1]}$ and that of the function $\psi'^*_x(t,x)$, imply that $P\{\int_0^1 \psi'^*_x(t,W_t(\omega))\|^2\,dt<\infty\}=1$. Therefore, for all $y^*\in Y^*$,

$$P\Big\{\int_0^1 \langle R\psi'^*_x(t,W_t)y^*, \psi'^*_x(t,W_t)y^*\rangle\,dt<\infty\Big\}$$

$$\geq P\Big\{\|R\|\,\|y^*\|^2\int_0^1 \Big\|\psi'^*_x(t,W_t(\omega)\Big\|^2\,dt<\infty\Big\}=1,$$

i.e., $\psi'^*_x(t,W_t)y^*\in\mathcal{L}_R(X^*)$. Also, if we denote

$$\varphi_n(t,\omega)\equiv\sum_{k=1}^{2^n}\psi'^*_x\Big(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}}\Big)\chi_{[\frac{k-1}{2^n},\frac{k}{2^n}]}(t),$$

then

$$P\Big\{\int_0^1 \Big\|\psi'^*_x(t,W_t)-\varphi_n(t,\omega)\Big\|^2\,dt\to 0 \quad\text{as}\quad n\to\infty\Big\}=1.$$

Therefore, for all $y^*\in Y^*$ and $\varepsilon>0$, as $n\to\infty$,

$$P\Big\{\int_0^1 \Big\langle R\Big(\psi'^*_x(t,W_t)-\varphi_n(t,\omega)\Big)y^*, \Big(\psi'^*_x(t,W_t)-\varphi_n(t,\omega)\Big)y^*\,dt\Big\rangle\varepsilon\Big\}\to 0.$$

Hence,

$$\sum_{k=1}^{2^n}\psi'^*_x(\frac{k-1}{2^n}, W_{\frac{k-1}{2^n}})y^*(W_{\frac{k}{2^n}}-W_{\frac{k-1}{2^n}})$$

converges in probability to

$$\int_0^1 \psi'^*_x(t,W_t)y^*\,dW_t$$

as $n\to\infty$.

Now, if we consider equality (1), act $y^*\in Y^*$ on its both sides, and pass to the limit in propability, we get

$$\Big\langle \psi(t,W_t)-\psi(0,0),y^*\Big\rangle=\int\limits_0^t\langle\psi_t'(s,W_s),y^*\rangle\,ds$$
$$+\int\limits_0^t\psi_x'(s,W_s)y^*\,dW_s+\frac{1}{2}\int\limits_0^t\Big\langle\psi_{xx}''(s,W_s)a_ia_i,y^*\Big\rangle\,ds. \tag{2}$$

It is clear that there exist $\int\limits_0^t\psi_t'(s,W_s)\,ds$ and $\int\limits_0^t\sum_i\psi_{xx}''(s,W_s)a_ia_i\,ds$ a.s., since

$$\int\limits_0^t\Big\|\sum_i\psi_{xx}''(s,W_s)a_ia_i\Big\|\,ds\le\int\limits_0^t\sum_i\|\psi_{xx}''(s,W_s)\|\,\|a_i\|^2\,ds$$
$$=\sum_i\|a_i\|^2\cdot\int\limits_0^t\|\psi_{xx}''(s,W_s(\omega))\|\,ds<\infty\quad\text{a.s.}$$

Thus, there exists a Y-valued random element

$$\xi_t\equiv\psi(t,W_t)-\psi(0,0)-\int\limits_0^t\psi_t'(s,W_s)\,ds-\frac{1}{2}\int\limits_0^t\sum_i\psi_{xx}''(s,W_s)a_ia_i\,ds$$

such that $\langle\xi_t,y^*\rangle=\int\limits_0^t\psi'^*_x(s,W_s)y^*\,dW_s$ a.s., i.e., there exists the stochastic integral $\int\limits_0^t\psi_x'(s,W_s)\,dW_s$ and the following equality is true

$$\psi(t,W_t)=\psi(0,0)+\int\limits_0^t\psi_t'(s,W_s)\,ds+\int\limits_0^t\psi_x'(s,W_s)\,dW_s$$
$$+\frac{1}{2}\int\limits_0^t\sum_i\psi_{xx}''(s,W_s)a_ia_i\,ds. \tag{3}$$

Above we have choosen $(a_k)_{k\in N}\subset X$ with $\sum\limits_i\|a_i\|^2<\infty$. Now it will be shown that in (3) we can choose an arbitrary $(a_k)_{k<N}$, participating in the representation of the covariance operator R.

Proposition 3. *Let (Ω',B',P') be another probability space, and $\xi(\omega')=\sum\limits_i a_i\gamma_i(\omega')$ be a Gaussian random element on it with a covariance operator R, $Rx^*=\sum\limits_{k=1}^{\infty}\langle a_k,\,x^*\rangle a_k$, where $(\gamma_i(\omega'))_{i\in N}$ are indipendent standard Gaussian random variables on (Ω',B',P'). Then*

$$\sum_i\psi_{xx}''(s,W_s(\omega))a_ia_i=E'\psi_{xx}''(s,W_s(\omega))\xi(\omega')\xi(\omega')$$

and the series in the left-hand side converges.

Proof. We have

$$E'\Big\|\psi''_{xx}(s,W_s(\omega))\xi(\omega')\xi(\omega')\Big\| \le \|\psi''_{xx}(s,W_s(\omega))\| \cdot E\|\xi(\omega')\|^2 < \infty.$$

Therefore, for any $y^* \in Y^*$,

$$E'\langle\psi''(s,W_s)\xi(\omega')\xi(\omega'),y^*\rangle$$
$$= E'\sum_{i,j}\langle\psi''(s,W_s(\omega))a_ia_j,y^*\rangle\gamma_i(\omega')\gamma_j(\omega') = \sum_i\langle\psi''(s,W_s(\omega))a_ia_i,y^*\rangle.$$

For any $k_n \uparrow \infty$ define $R_1x^* = \sum\limits_{n=1}^{\infty}\langle a_{k_n},x^*\rangle a_{k_n}$. Then

$$\langle R_1x^*,x^*\rangle = \sum_{n=1}^{\infty}\langle a_{k_n},x^*\rangle^2 \le \sum_{n=1}^{\infty}\langle a_n,x^*\rangle^2 = \langle Rx^*,x^*\rangle.$$

Therefore R_1 is a Gaussian covariance too (see [4], p. 310) and as shown above, for all $y^* \in Y^*$,

$$\sum_n\langle\psi''_{xx}(s,W_s(\omega))a_{k_n}a_{k_n},y^*\rangle$$

converges to $E'\langle\psi''_{xx}(s,W_s(\omega))\xi_1(\omega')\xi_1(\omega'),y^*\rangle$, where ξ_1 is a Gaussian random element with the covariance operator R_1. By the Orlicz–Pettis theorem ([11], p. 24) $\sum\limits_k \psi''_{xx}(s,W_s(\omega))a_ka_k$ converges, and the sum $E'\psi''_{xx}(t,W_t)\xi(\omega')\xi(\omega')$ does not depent on the sequence $(a_k)_{k\in N}$. Proposition 3 is proved. Hence, Theorem 1 is also proved. □

If Y is a separable Hilbert space, then the result of Theorem 1 can be found in [8]. If Y is a separable Banach space, where the square of the norm has a derivative satisfying the Lipschitz condition, then our result is contained in [9].

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